BOSCH

Task 2: spherical harmonics overlap

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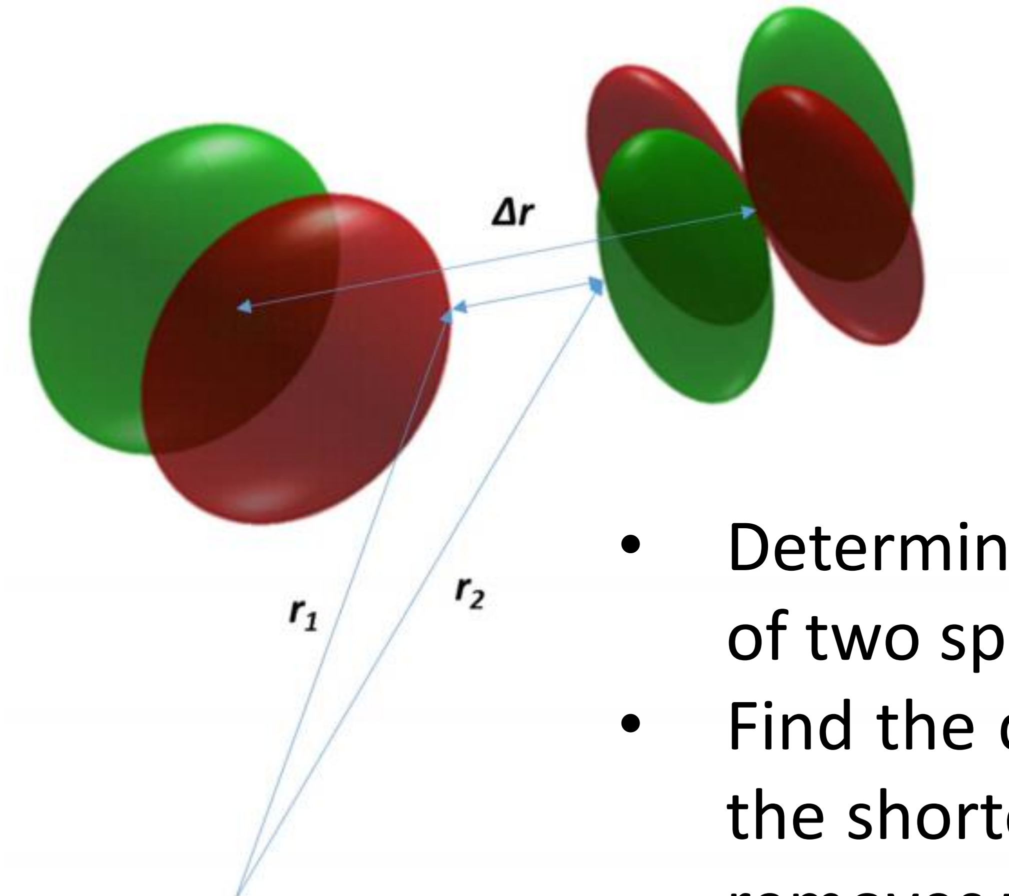
Task formulation



- Determine the closest points of two spherical harmonics.
- Find the degree of overlap, i.e. the shortest shift vector, which removes the overlap.

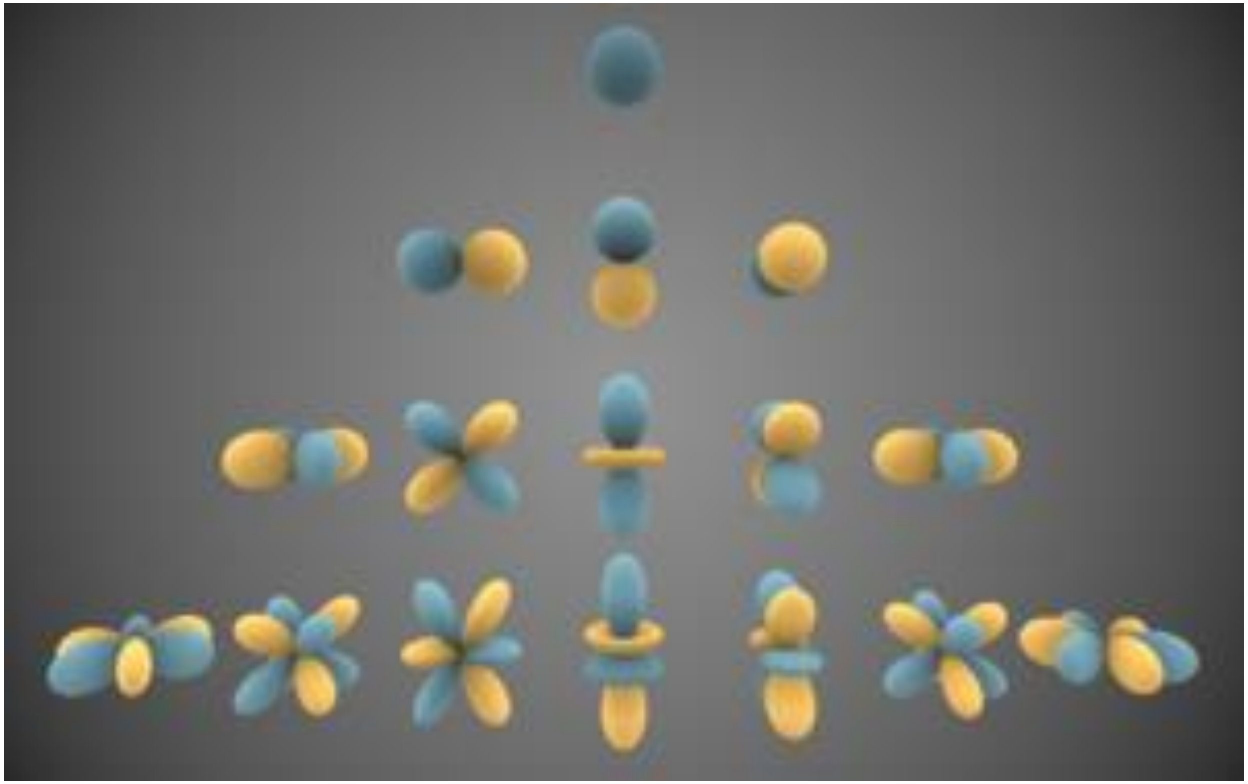
Spherical harmonics

$$Y_l^m(\varphi, \vartheta) = \frac{1}{\sqrt{2\pi}} e^{im\varphi} P_l^m(\cos\vartheta), \qquad 0 \le |m| < l$$

$$P_l^m(\cos\vartheta) = (-1)^m (\sin\vartheta)^m \frac{d^m}{d(\cos\vartheta)^m} (P_l(\cos\vartheta))$$

$$P_l(\cos\vartheta) = \frac{1}{2^l l!} \frac{d^l}{d(\cos\vartheta)^l} (\cos^2\vartheta - 1)^l$$

Spherical harmonics

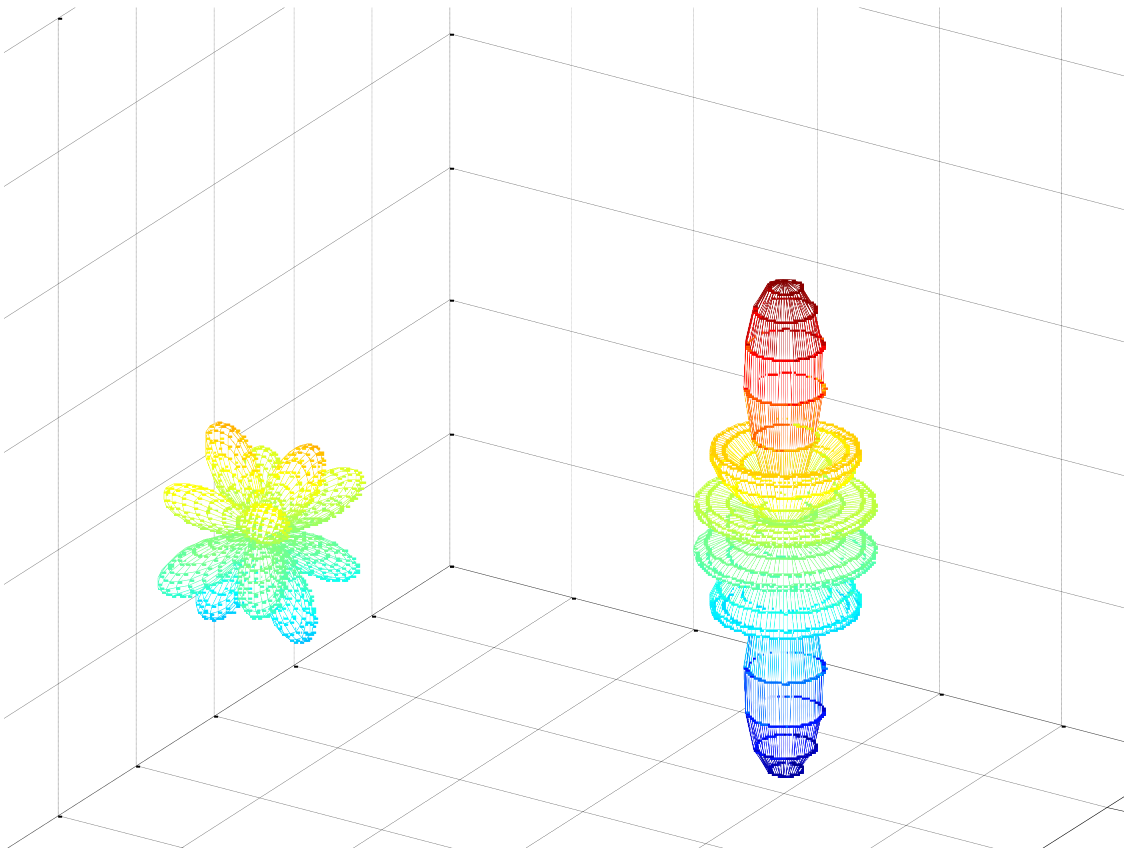


$$r_l^m(\varphi,\vartheta) = \left|Re\left(Y_l^m(\varphi,\vartheta)\right)\right| = \frac{1}{\sqrt{2\pi}}|\cos(m\varphi)|P_l^m(\cos\vartheta)$$

Approach

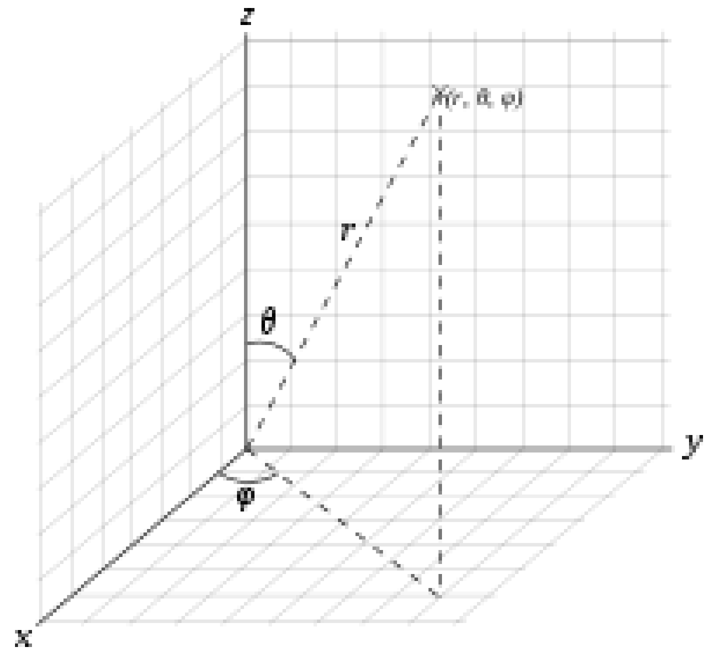
- Numerical approach:
  - separate optimization problems for intersecting and non-intersecting cases.
- Geometry based approach:
  - optimization based on hyperplanes and surface normals.
- Analytical approach:
  - estimate distance and shift vector by covering the parts of the harmonics with spheres;
  - create different basis from convex shapes.

Numerical algorithm: distance



Formalization and solution

Spherical coordinate system



$$\min_{\substack{\overrightarrow{r_1} \in F_1 \\ \overrightarrow{r_2} \in F_2}} |\overrightarrow{r_1} - \overrightarrow{r_2}| =$$

$$= \min_{\substack{\varphi_1, \varphi_2 \in [0, 2\pi] \\ \vartheta_1, \vartheta_2 \in [0, \pi]}} |\overrightarrow{r_1}(\varphi_1, \vartheta_1) - \overrightarrow{r_2}(\varphi_2, \vartheta_2)|$$

0-order method (coordinate descent)

different results for each pair of parts:

different initial values

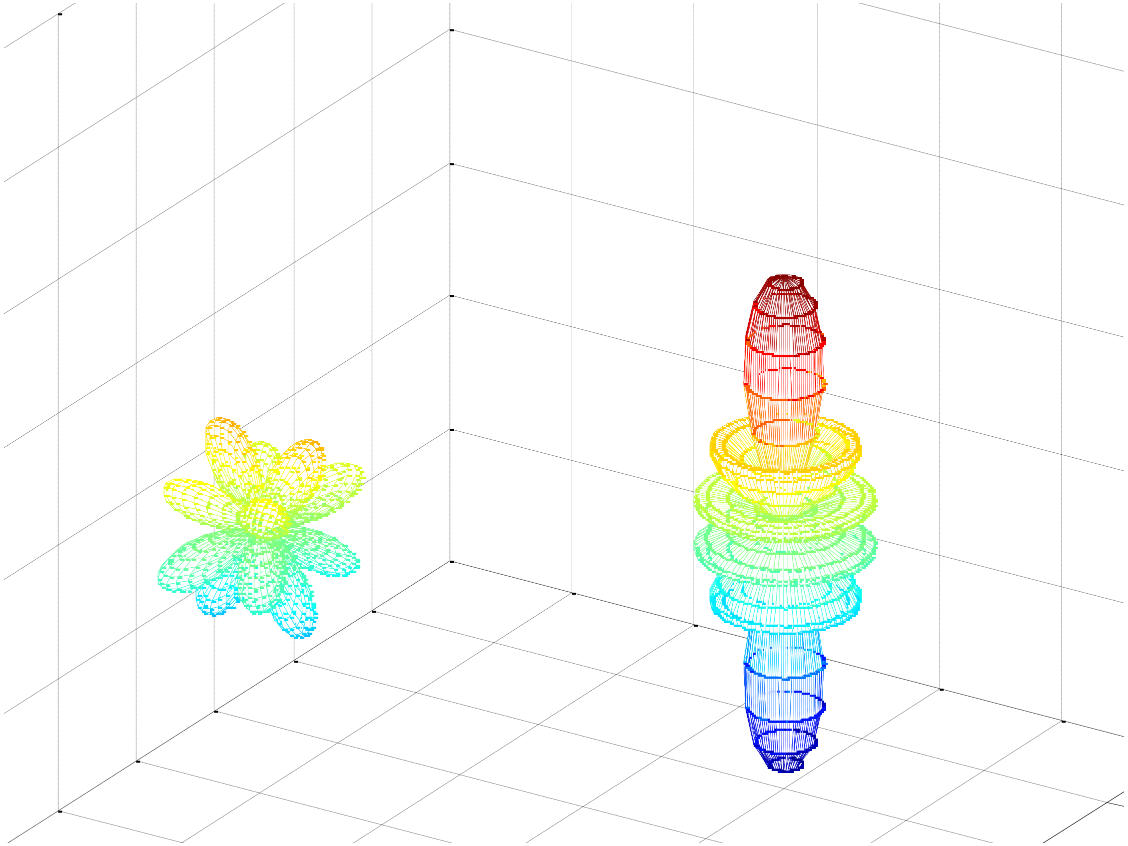
Splitting harmonics into parts

Find one point for each part

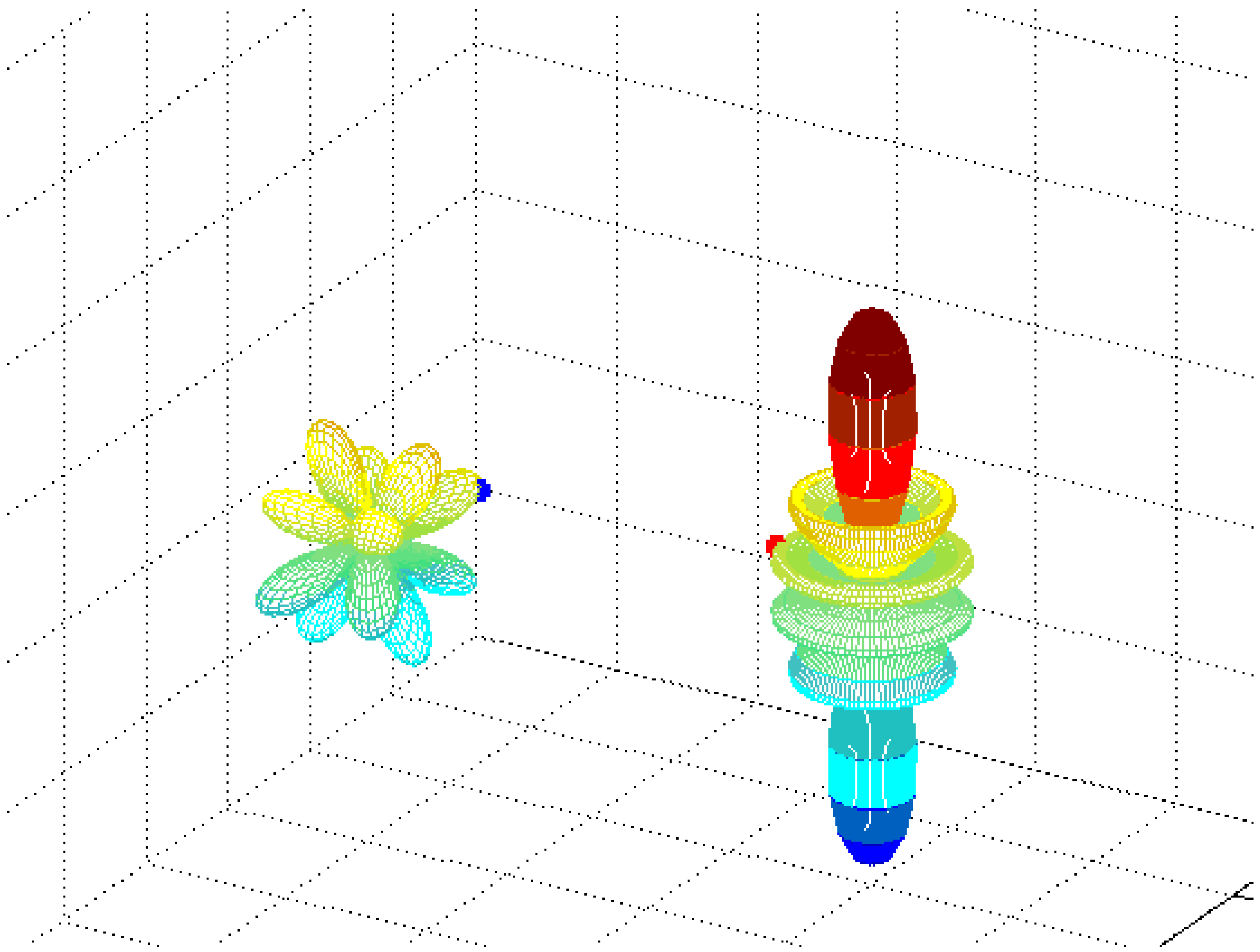
$$\underset{\substack{\varphi\in[0,2\pi]\\ \vartheta\in[0,\pi]}}{\operatorname{argmax}}\left|Re\left(Y_l^m(\varphi,\vartheta)\right)\right|$$

- $|\cos m\varphi_k|' = 0$
  - $m \neq 0 \Rightarrow \varphi_k = \frac{\pi k}{m} + \frac{\pi}{2m}, k \in [0,2m-1]$
  - $m = 0 \Rightarrow \varphi_0 = 0$
- $\left(P_l^m(\cos\vartheta)\right)' = 0 \Rightarrow \vartheta_k$

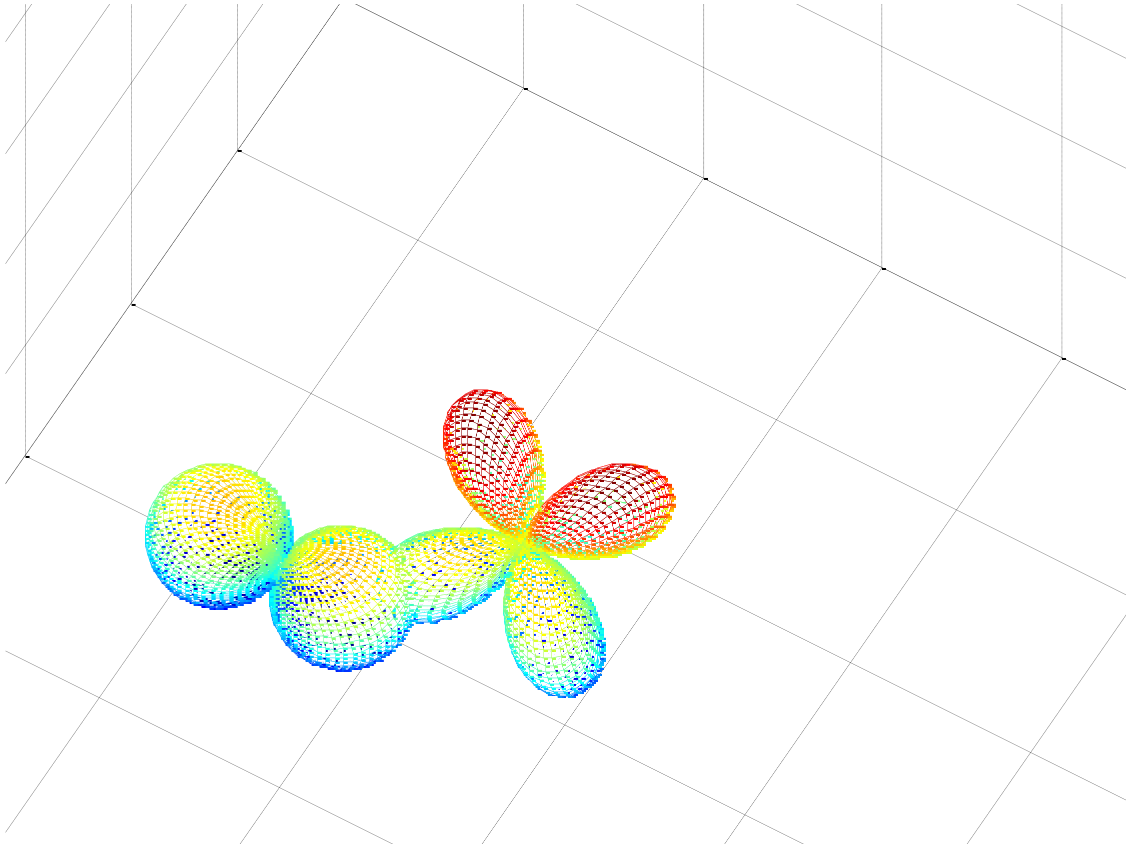
Numerical algorithm: distance



Numerical algorithm: distance (result)

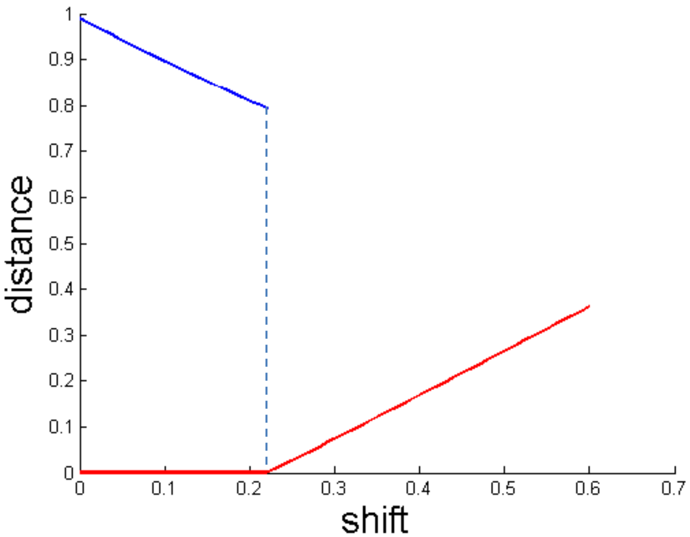


Numerical algorithm : intersection



Formalization and solution

$$\min_{\substack{\vec{s}\in\mathbb{R}^3 \\ figures\ do\ not\ intersect \\ after\ shift\ \vec{s}}} |\vec{s}| = \min_{\substack{\vec{s}\in\mathbb{R}^3 \\ \rho(F_1,F_2+\vec{s})>0}} |\vec{s}|$$



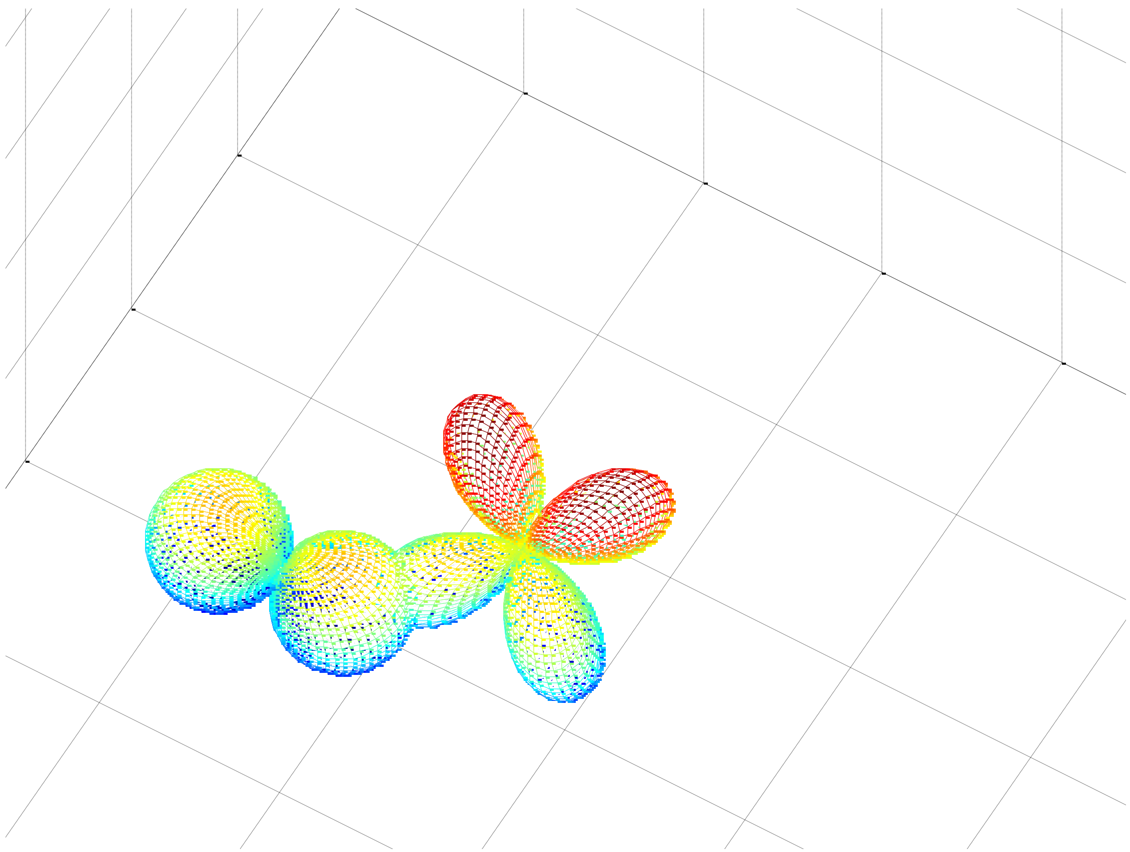
Nonlinear constraint optimization problem

Penalty method:

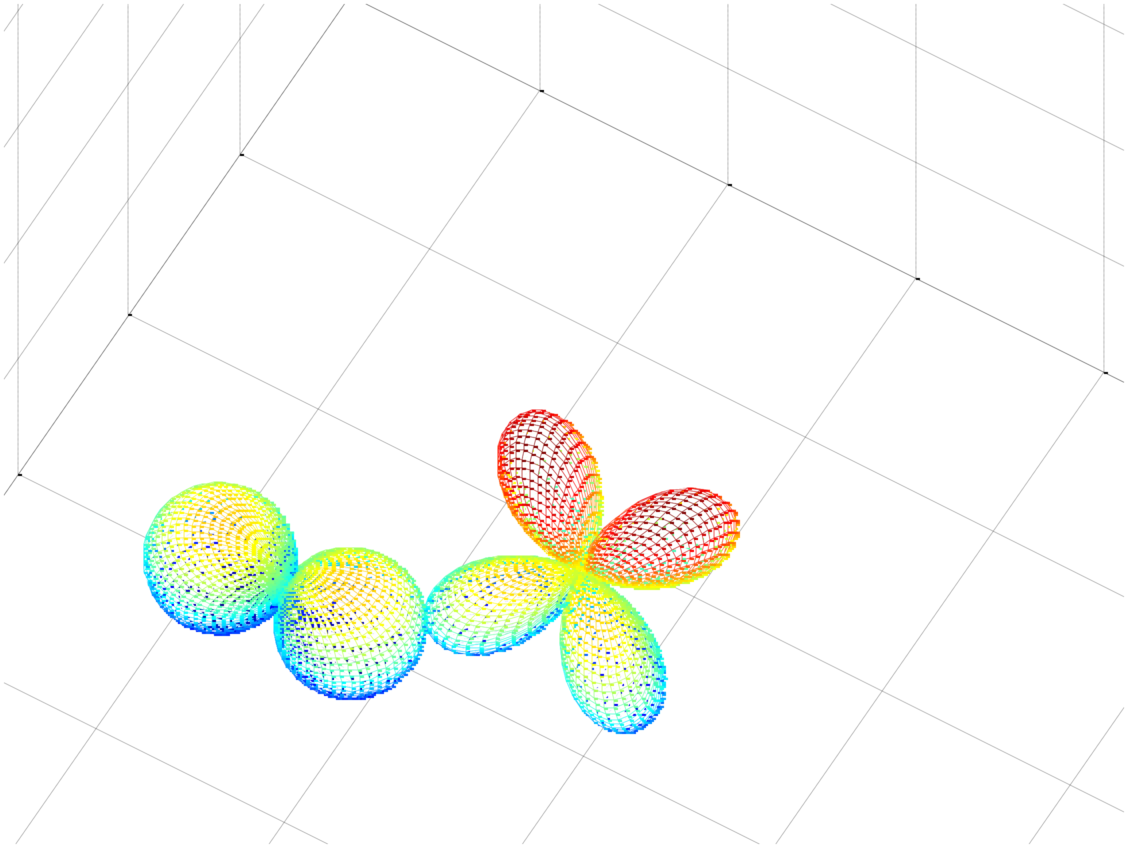
$$\rho(F_1, F_2 + \vec{s}) < \varepsilon \Rightarrow |\vec{s}| := Me^{-|\vec{s}|}$$

$$\overrightarrow{s_0} = \Delta\vec{r}$$

Numerical algorithm: intersection



Numerical algorithm: intersection (result)

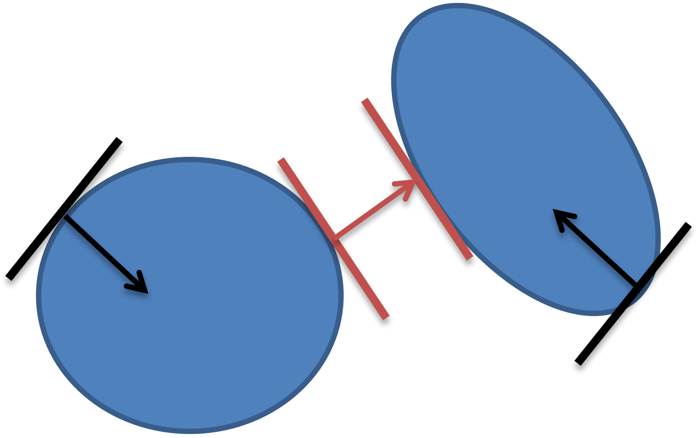


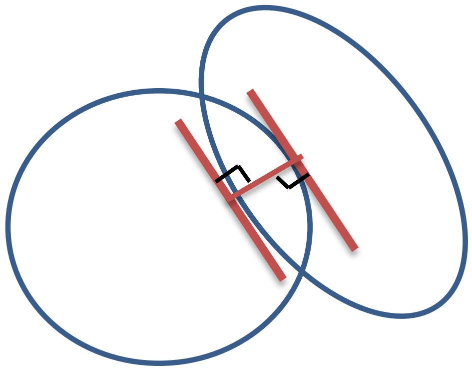
Geometry based approach

$$
\begin{cases}
\min\limits_{x_1,x_2,y_1,y_2,z_1,z_2} \left(\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}\right) \\
\dfrac{x_1 - x_2}{\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial x}(x_1)} + \dfrac{y_1 - y_2}{\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial y}(y_1)} = 0 \\
\dfrac{y_1 - y_2}{\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial y}(y_1)} + \dfrac{z_1 - z_2}{\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial z}(z_1)} = 0
\end{cases}
$$

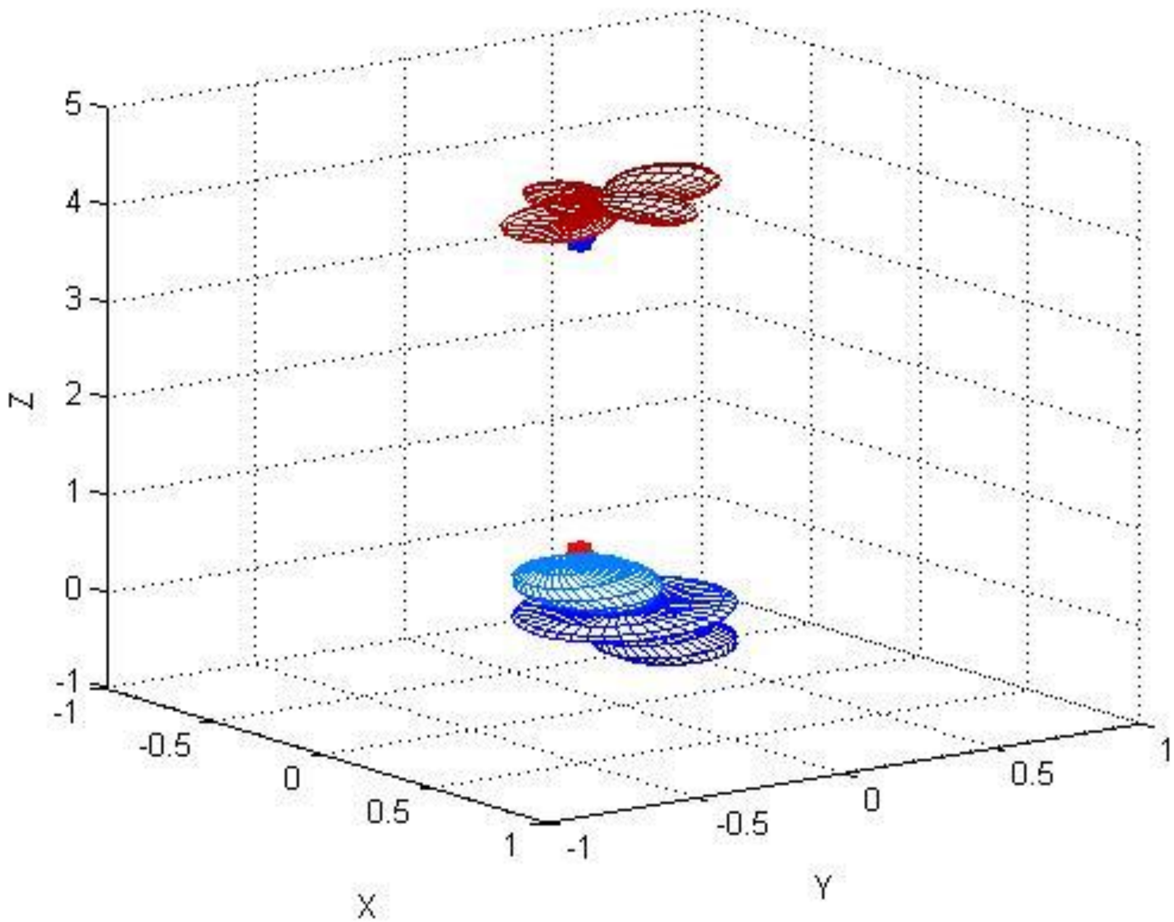
Geometry based approach

$$\begin{cases} \min\limits_{x_1,x_2,y_1,y_2,z_1,z_2}\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}\right) \\ \dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial x}(r_1) = -\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial x}(r_2) \\ \dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial y}(r_1) = -\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial y}(r_2) \\ \dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial z}(r_2) = -\dfrac{\partial Y_l^m(\varphi,\vartheta)}{\partial z}(r_2) \end{cases}$$

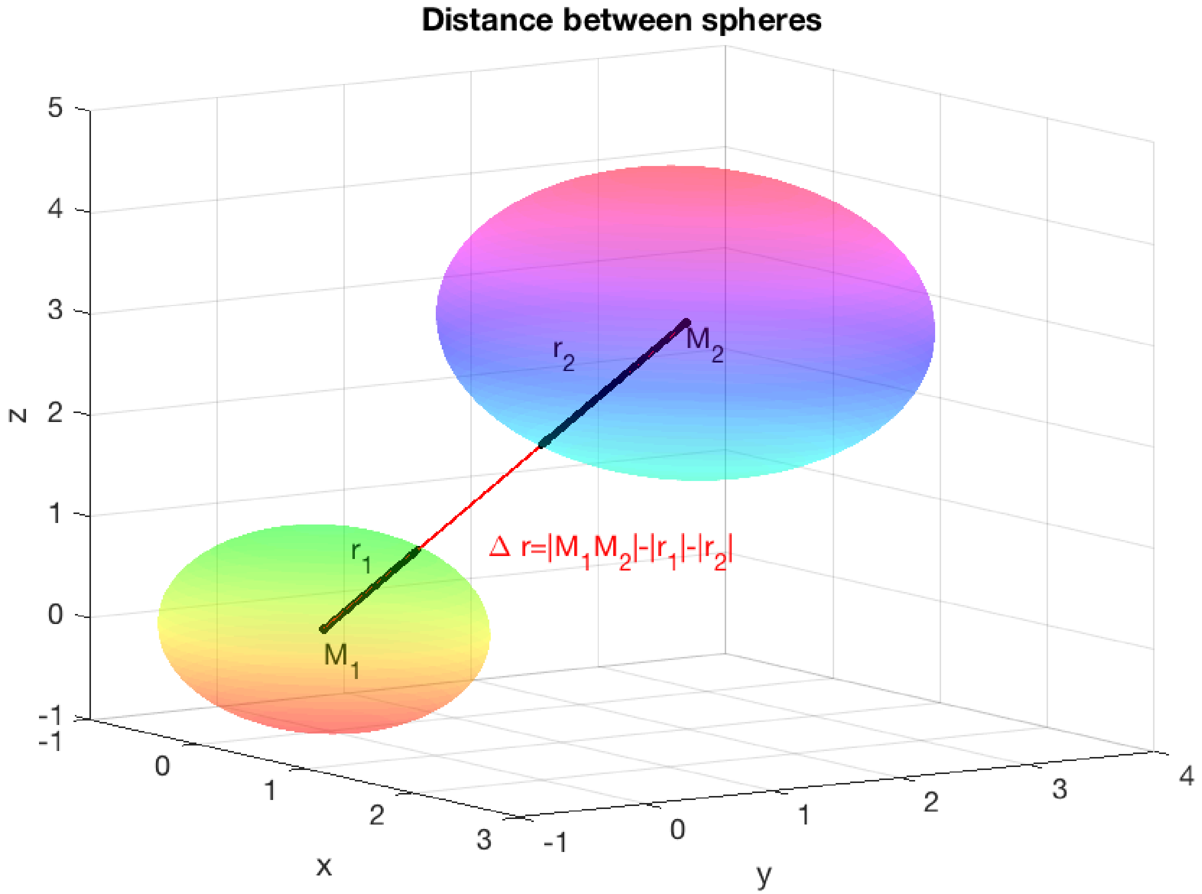




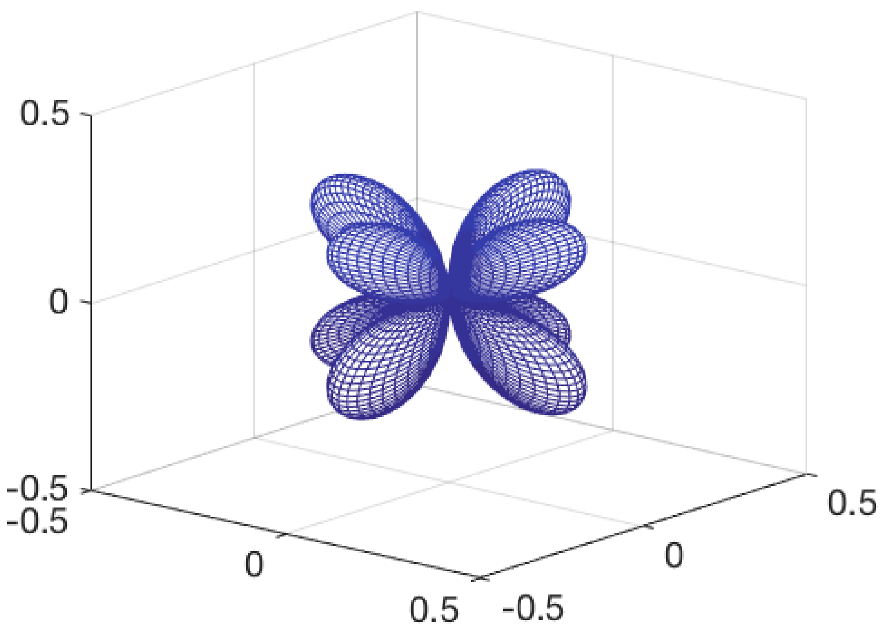
Geometry based approach: preliminary result



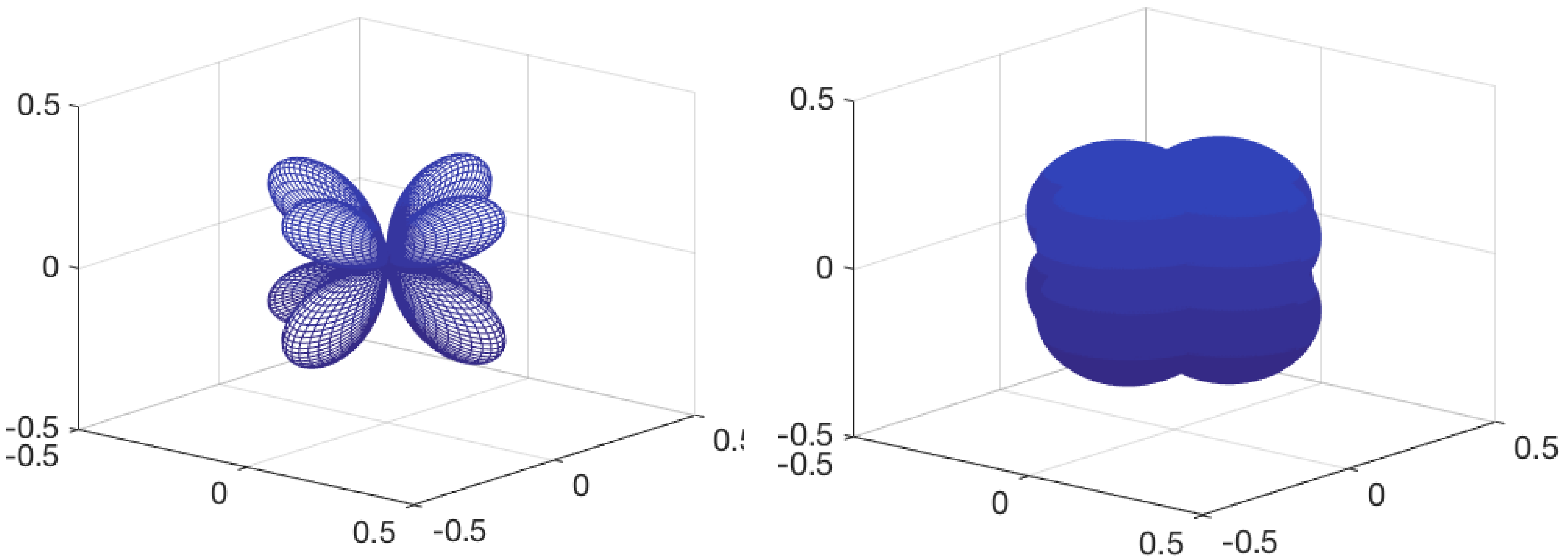
Analytical approach



First idea: cover-up with spheres

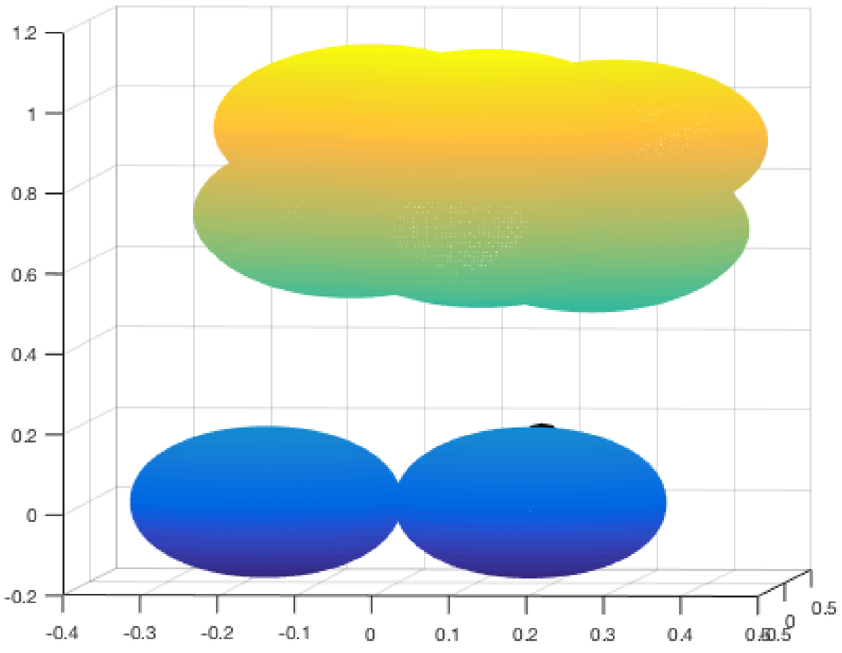


First idea: cover-up with spheres



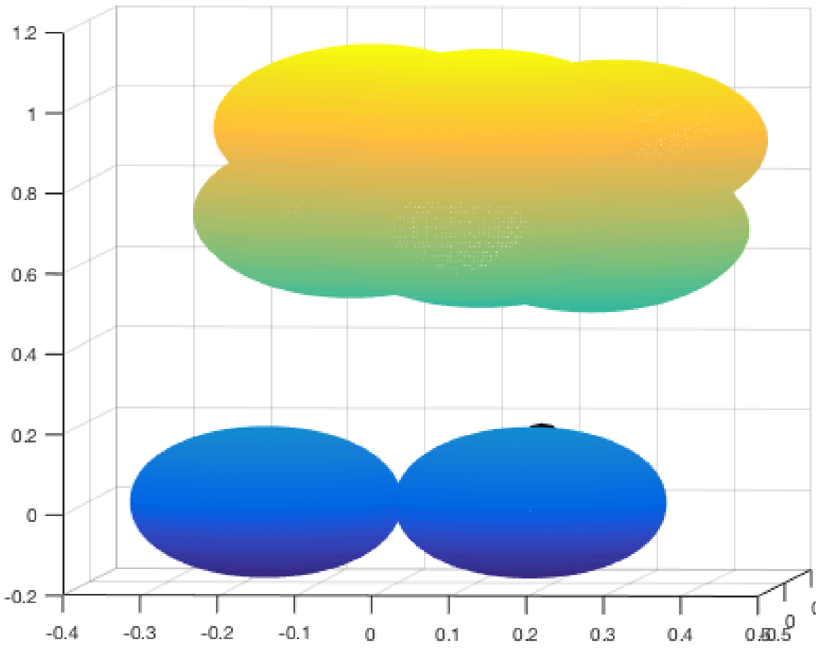
First idea: cover-up with spheres

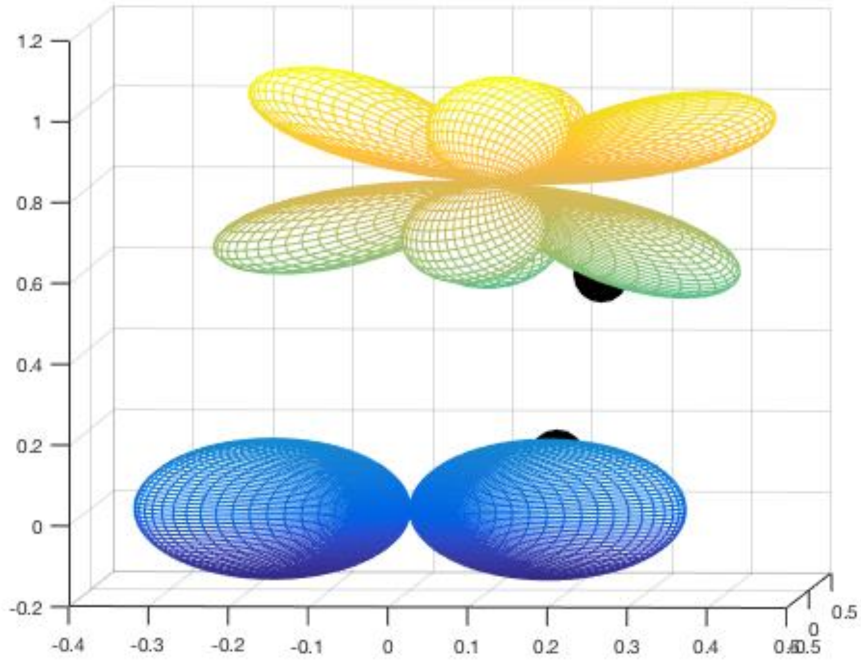
- Calculate closest distance analytically



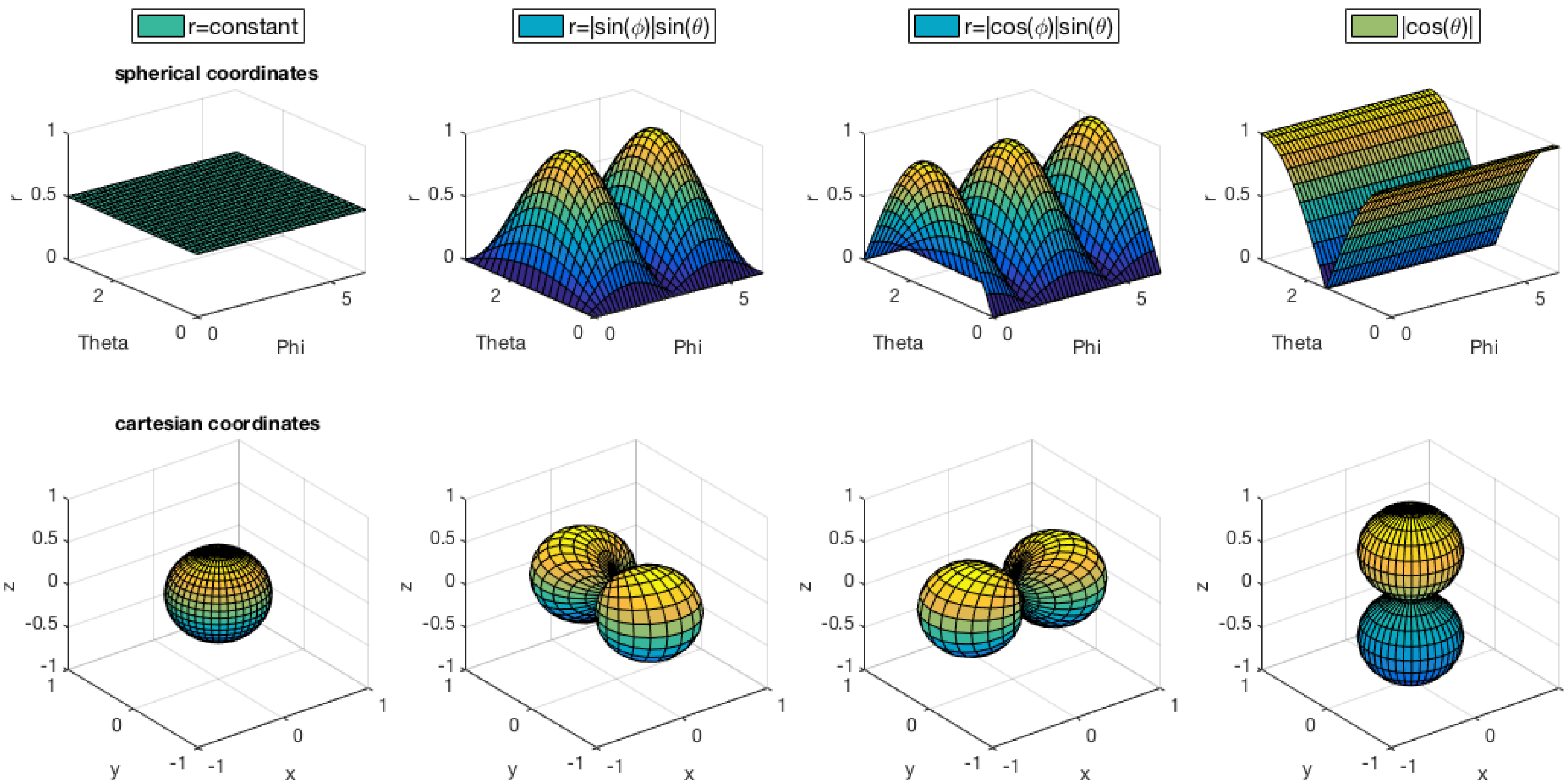
First idea: cover-up with spheres

- Calculate closest distance analytically
- Obtain lower bound for the real distance

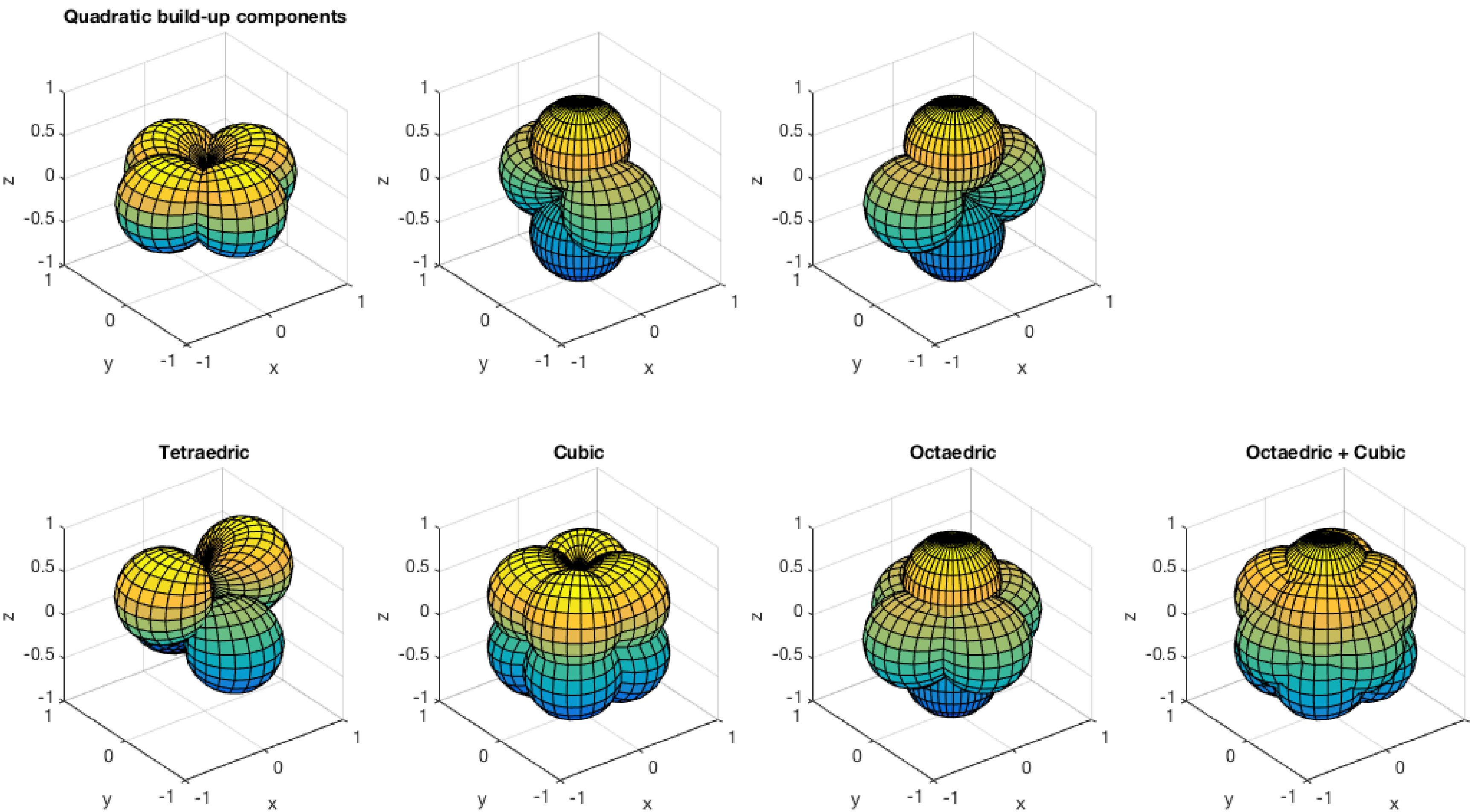




Second idea: create spherical basis



Some generated shapes



Results

- Numerical approach
  - Implemented Matlab algorithm
- Geometry based approach
  - Developed algorithm
- Analytical approach
  - Allows fast analytic computation
  - Creation of new spherical basis

Thank YOU!

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